JUNE 2019 PAPER 3

1. **A company which employs 10 unskilled and 8 skilled workers produces two models of a toy, Model A and Model B.**
   - **Model A requires 20 hours of unskilled labour while Model B requires 15 hours of unskilled labour**
   - **Model A requires 10 hours of skilled labour while Model B requires 25 hours of skilled labour**

   Each worker provides 40 hours of labour per week.

(a) Calculate the maximum number of hours available per week for (i) unskilled labour (ii) unskilled labour

SOLUTION:
Data: The company employs 10 unskilled workers and each worker provides 40 hours of labour per week. The company also employs 8 skilled workers and each worker provides 40 hours of labour per week.

Required to calculate: the maximum number of hours available for (i) unskilled labour and (ii) skilled labour.

Calculation:
Maximum number of hours available for unskilled labour $= 40\ hours \times 10$
$= 400\ hours$

Maximum number of hours available for skilled labour $= 40\ hours \times 8$
$= 320\ hours$

(b) **Assume that the factory makes x Model A and y Model B toys per week. Show that The inequalities for the hours of labour are:**

(i) unskilled labour, $4x + 3y \leq 80$ (ii) skilled labour, $2x + 5y \leq 64$.

SOLUTION:
Data:
The factory makes x Model A and y Model B toys per week
Model A requires 20 hours of unskilled labour while Model B requires 15 hours of unskilled labour.
Model A requires 10 hours of skilled labour while Model B requires 25 hours of skilled labour.

Required to show: the inequalities for the hours of labour are:
(i) unskilled labour: $4x + 3y \leq 80$ and (ii) skilled labour: $2x + 5y \leq 64$.

Solution:
The information can be represented in the following table:

	No. of hours		Maximum number of hours
	Model A (x)	Model B (y)	
Unskilled labour	20	15	400
Skilled labour	10	25	320

(i) Unskilled labour:
Each Model A toy require 20 hours of unskilled labour
x Model A toys will require $20x$ hours of unskilled labour

Each Model B toy require 15 hours of unskilled labour
y Model A toys will require $15y$ hours of unskilled labour

Total number of hours required for unskilled labour is: $20x + 15y$
The maximum number of hours available is 400.
Therefore, the inequality that represents this condition is: $20x + 15y \leq 400$

Dividing by 5, this simplifies to: $4x + 3y \leq 80$

(ii) Skilled labour:
Each Model A toy require 10 hours of skilled labour
x Model A toys will require $10x$ hours of skilled labour

Each Model B toy require 25 hours of skilled labour
y Model A toys will require $25y$ hours of skilled labour

Total number of hours required for skilled labour is: $10x + 25y$
The maximum number of hours available is 320.
Therefore, the inequality that represents this condition is: $10x + 25y \leq 320$

Dividing by 5, this simplifies to: $2x + 5y \leq 64$

(c) **Show, by calculation that it is possible to make 15 Model A toys and 5 Model B toys.**

SOLUTION:
Data: For x Model A toys and y Model B toys, the following requirements for hours of labour must be satisfied:
Unskilled labour: $4x + 3y \leq 80$ Skilled labour: $2x + 5y \leq 64$

Required: To show by calculation that it is possible to make 15 Model A toys and 5 Model B toys.
Calculation:
The number of Model A toys $= x = 15$ and the number of Model B toys $= y = 5$

For unskilled labour, $4x + 3y \leq 80$ So, $4(15) + 3(5) = 60 + 15 = 75$ Since $75 \leq 80$, we conclude that it is possible to make 15 Model A toys and 5 model B toys.	For skilled labour, $2x + 5y \leq 64$ So, $2(15) + 5(5) = 30 + 25 = 55$ Since $55 \leq 64$, we conclude that it is possible to make 15 Model A toys and 5 model B toys.

Note also, we could have substituted these values in the original inequalities to obtain the same conclusion. The calculations are shown below:

For unskilled labour, $20x + 15y \leq 400$ So, $20(15) + 15(5) = 300 + 75 = 375$ Since $375 \leq 400$, we conclude that it is possible to make 15 Model A toys and 5 model B toys.	For skilled labour, $10x + 25y \leq 320$ So, $10(15) + 25(5) = 150 + 125 = 275$ Since $275 \leq 320$, we conclude that it is possible to make 15 Model A toys and 5 model B toys.

(d) **The factory makes a profit of \$40 on a Model A toy and \$60 on a Model B toy each week.**

(i) Write an equation for the TOTAL profit, *P*, per week, for making *x* Model A and *y* Model B toys.

SOLUTION:
Data: The profit per week on one Model A toy is \$40 and the profit on one Model B toy is \$60.
Required: To write an equation for the Total profit made per week on *x* model A toys and *y* Model B toys.
Profit Equation:
Profit per week on one Model A toy is \$40. Profit per week on *x* Model A toys is $\$40x$
Profit per week on one Model B toy is \$60. Profit per week on *y* Model B toys is $\$60y$
Total profit per week on both type of toys in dollars is : $P = 40x + 60y$

(ii) What is the profit to the company when it makes 15 Model A toys and 5 Model B toys?

SOLUTION:
Data: The total profit, *P*, per week on *x* Model A toys and *y* Model B toys is $40x + 60y$.

Required: To calculate the profit per week made on 15 model A toys and 5 Model B toys.
Calculation:

$$P = 40x + 60y$$

Substitute, $x = 15$, and $y = 5$

$$P = \$40(15) + \$60(5) = \$600 + \$300 = \$900$$

(e) Show that the profit calculated in (d) (ii) is NOT the maximum profit.

SOLUTION:
Data: Total profit per week on both type of toys in dollars is : $P = 40x + 60y$
Required: To show that the profit of $900 made on 15 model A toys and 5 Model B toys is not the maximum profit.
Solution:

In order to show that the profit calculated when $x = 15$ and $y = 5$ is not the maximum, we must find a pair of values for *x* and *y* that would result in a profit that is more than $900.
Consider the point, $x = 14$ and $y = 6$. We need to calculate the profit on making 14 Model A toys and 6 Model B toys.
Before doing so, we must first test to see that this point satisfies the given conditions:

$x \geq 0, y \geq 0, 4x + 3y \leq 80$ and $2x + 5y \leq 62$

Letting $x = 14$ and $y = 6$, we note that, $14 \geq 0$ and $6 \geq 0$
Also, $4(14) + 3(6) = 56 + 18 = 74 \leq 80$
And, $2(14) + 5(6) = 28 + 30 = 58 \leq 62$

Since (14, 6) satisfies all conditions and lies within the feasible region, it is possible to make 14 Model A toys and 6 Model B toys.

The profit on 14 Model A toys and 6 Model B toys is:

$$P = 40x + 60y$$

Substitute, $x = 14$, and $y = 6$

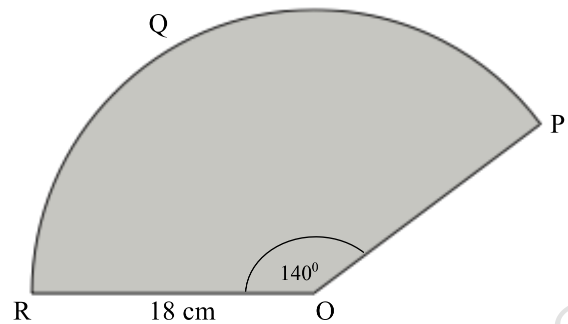
$$P = \$40(14) + \$60(6) = \$560 + \$360 = \$920$$

Since $\$920 > \$900$, the maximum profit cannot be $900.

Q.E.D.

2. **The diagram below, not drawn to scale, shows a flexible piece of card in the shape of a sector of a circle with centre *O* and radius 18 cm.**

Use $\pi = \frac{22}{7}$



(a) **Show that the perimeter of the card is 80 cm.**

SOLUTION:
Data: Diagram showing a card in the shape of a sector of a circle of radius 18 cm and angle at the centre equal to 140^0.
Required to show: The perimeter of the sector is 80 cm.
Proof:
Perimeter of the sector = length of arc RQP + length of radius PO + length of radius OR

Arc length RPQ $= \frac{\theta}{360} \times 2\pi r$, where θ is the angle in the sector and *r* is the radius of the circle.

Arc length RPQ $= \frac{140}{360} \times 2 \times \frac{22}{7} \times 18 = 44$ cm

Since PO and OR are radii of the circle, PO=OR= 18 cm
Perimeter of the sector = Arc length RQP + PO + OR
= 44 + 18 + 18 cm
= 80 cm

Q.E.D.

(b) **Calculate the area of the card *OPQR*.**

Required to calculate: The area of the card *OPQR*
Calculation:

Area of sector $= \frac{\theta}{360} \times \pi r^2$

Area of card $= \frac{140}{360} \times \frac{22}{7} \times 18 \times 18 = 396\ cm^2$

(c) **The card is bent and the edges OP and OR are taped together so that the card forms the curved surface of a cone with a circular base, PQR.**

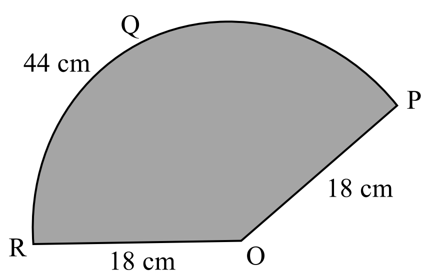
(i) **Draw a diagram of the cone formed, showing clearly the measurement 18 cm, the perpendicular height, *h*, and the radius, *r*, of the base of the cone.**

SOLUTION:

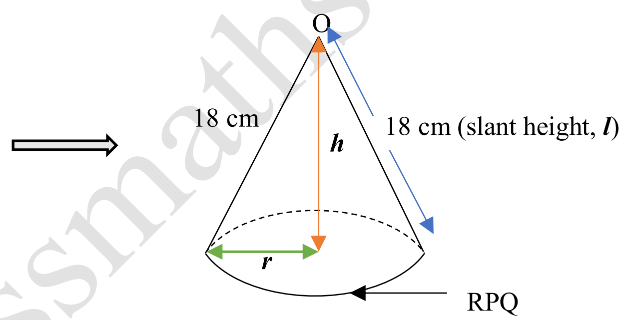
Data: A diagram of a sector of a circle with radius 18 cm.

Required: To draw a diagram of the cone formed when the card is taped.

Drawing:

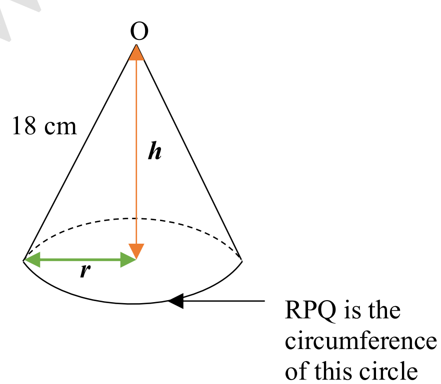


Sector or net of the cone. When folded OP and OQ will coincide.



Cone – the arc of the sector, RPQ becomes its circular base. The radius OP or OR becomes the slant height, ***l***. The distance from the apex of the cone to the center of its circular base, ***h***, is the perpendicular height and ***r*** is the radius of the cone.

(ii) Calculate the radius of the circular base of the cone.



Circumference of the circular base of the cone, RPQ = 44 cm

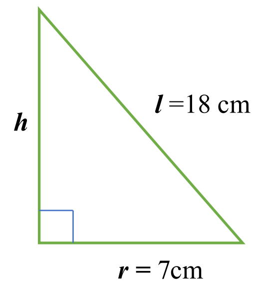
$$2\pi r = 44$$
$$2 \times \frac{22}{7} \times r = 44$$
$$r = \frac{44 \times 7}{2 \times 22} = 7\ cm$$

Radius of base of cone = 7 cm

(iii) **Using Pythagoras' theorem, or otherwise, determine the perpendicular height of the resulting cone**.

SOLUTION:

Data: The radius, $r = 7$ cm and slant height, $l = 18$ cm of a cone.
Required: To calculate the perpendicular height of the cone.
Calculation:



By Pythagoras' Theorem

$h^2 + r^2 = l^2$

$h^2 \quad = l^2 - r^2$

$h^2 \quad = l^2 - r^2$

$h^2 \quad = 18^2 - 7^2 = 324 - 49$

$h^2 \quad = 275$

$h \quad = \sqrt{275} = \sqrt{25 \times 11} = 5\sqrt{11}$ (in exact form)

$h = 16.58$ cm (correct to 2 decimal places)